MCR3U FINAL EXAM REVIEW (JANUARY 2015)

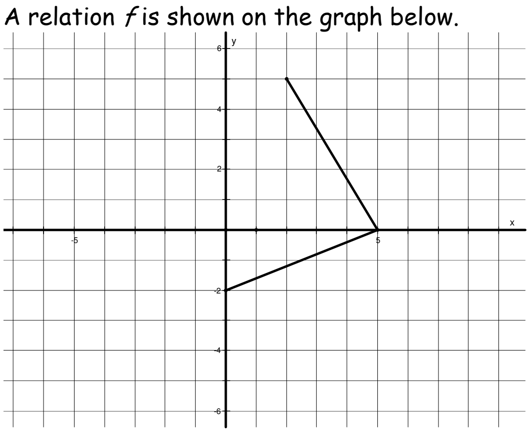
Introduction: This review is composed of possible test questions. The BEST way to study for math is to do a wide selection of questions. This review should take you a total of 6 hours of work, provided you can refer to your notes easily for questions you have difficulty with. Once you are done, you will have an inventory of all possible types of exam questions.

1. Consider the following sets:
$$f=\{(2,1),(3,1),(4,1)\}$$
$$g=\{(1,2),(1,3),(1,4)\}$$
$$h=\{(1,5),(2,6),(3,7),(4,2)\}$$
   (a) State the domain of f.
   (b) Which of the sets are functions?
   (c) State the range of g.
   (d) State the value of $h(2)$
   (e) Is it true that $f^{-1}=g$? Explain.
   (f) $h(x) = 7$. State the value of x
   (g) State the value of $h^{-1}(2)$
   (h) Create a mapping diagram for h.
   (i) True or false: the domain of g^{-1} is the same as the range of f.

2. State the domain and range for the function whose defining equation is $y=-2(x-1)^2+6$. Explain how you know.

3. Determine the inverse of the following functions. Then state the domain and range of both the original and its inverse.
   (a) $f(x)=2x+9$
   (b) $f(x)=\sqrt{x+4}-5$

4. A relation f is shown on the graph below.



   (a) Is the relation a function? Explain.
   (b) Evaluate $f(0)$
   (c) Determine the value of x such that f(x) = 5.
   (d) State the domain and range of f
   (e) Graph the line y = x and the inverse, f^{-1}, on the same grid as the relation f.

5. Suppose that $f(x)=2x-1$ and $g(x)=x^2+3$
   (a) Determine $g(-3)$
   (b) Determine $g(x-3)$, and fully simplify your answer.
   (c) Determine $f(7)$.

6. Graph the following base functions on graph paper. Be sure to show an appropriate number of key points. Graph any asymptotes with a dotted line and label with its equation.
   (a) $y=x^2$ (b) $y=\sqrt{x}$ (c) $y=\frac{1}{x}$ (d) $y=2^x$

7. Describe in words, using vocabulary suitable for this course, how
   (a) the graph of $y=3\sqrt{-2x+8}+11$ is transformed from the graph of $y=\sqrt{x}$.
   (b) the graph of $y=-\frac{1}{5}(3)^{0.5x+1}-9$ is transformed from the graph of $y=(3^x)$.

8. State the equations of the asymptotes for the following rational and exponential functions.
   (a) $y=\frac{1}{x}$
   (b) $y=\frac{2}{x+5}-6$
   (c) $y=-\frac{3}{x-8}+7$
   (d) $y=(4)^x$
   (e) $y=\frac{1}{3}(4)^{-2x}-9$
   (f) $y=-5(4)^{x+7}+12$

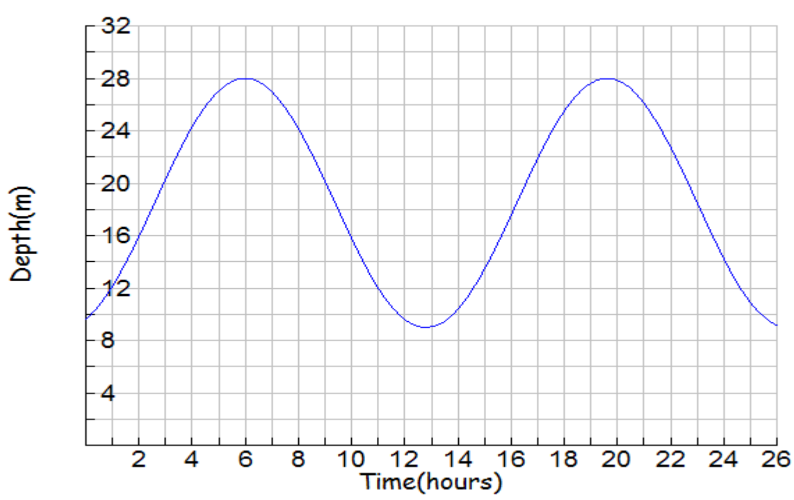
9. Sketch the following functions on graph paper. Draw any asymptotes with a dotted line and label with its equation.
   (a) $y=2(x+4)^2-3$
   (b) $y=-\left(\frac{1}{x-1}\right)+3$
   (c) $y=-4\sqrt{-(x+2)}+5$

10. The graph of $y = \sin(x)$ is reflected in the y-axis, vertically stretched by a factor of 3 and translated 4 units up and 2 units right. What is the equation of the new graph?

11. Complete the chart for the following sinusoidal functions. Then graph y = sinθ, y = 3sin[1/2(θ – 60°)] – 2, and y = -1.5sin[3(θ + 240°)] + 5.5 on the same grid over 2 cycles.

Function	**Amplitude**	**Period**	**Max y**	**Min y**	**Phase shift**	**Vertical Shift**	**Domain**	**Range**
$y = \sin\theta$								
$y = 3\sin\left[\frac{1}{2}(\theta - 60°)\right] - 2$								
$y = -1.5\sin[3(\theta + 240°)] + 5.5$								
$y = 5\cos[4(\theta - 22°)] + 8$								
$y = -2\sin[5(\theta + 40°)] + 15$								
$y = -8\cos\left[\frac{1}{5}\theta\right] - 11$								

12. The graph below depicts the depth of the water on a typical day at ocean port.

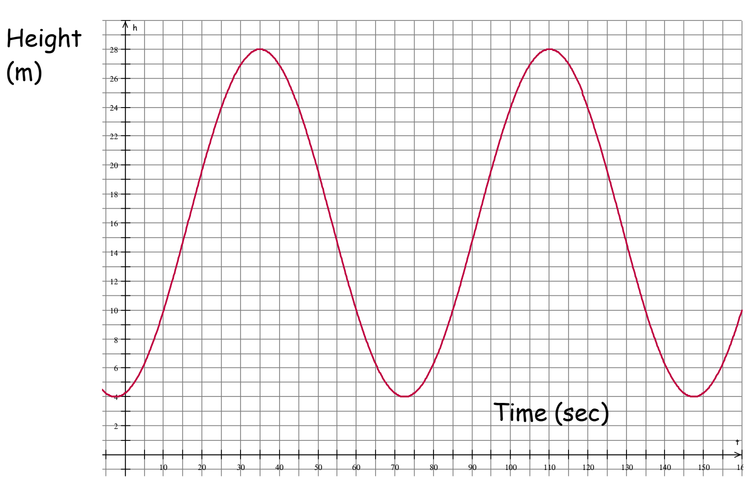


a) Determine the maximum and minimum depths of the water.

b) State the amplitude, period, and vertical displacement for this relationship.

c) Assuming the graph is a transformed cosine graph, determine a possible equation for the graph.

13. A Ferris Wheel takes 35 seconds from its embarkment point to reach its maximum height of 28m above the ground. The Ferris Wheel's minimum height of 4m above the ground occurs 37.5 seconds later. Below is the sinusoidal curve that models this scenario.



a) Determine an equation that models the above scenario for time, t, in seconds and height, h, in meters, in the form $h = a\cos[k(t - d)] + c$.

b) Use the equation to determine the height of the Ferris Wheel after 58 seconds, to the nearest tenth of a meter.

14. Solve the following triangles. Don't forget to check for the AMBIGUOUS CASE, where necessary. Round angle measures to the nearest degree and side lengths to one decimal place.

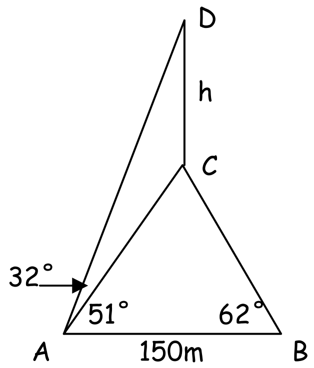
a) ΔABC where ∠B = 109°, a = 8cm, and c = 6cm.

b) ΔDEF, where d = 3.2cm, f = 5.6cm and ∠D = 22°.

c) ΔGHI, where ∠G = 25°, ∠H = 62°, and i = 4.8cm.

d) ΔJKL, where j = 4cm, l = 5cm, and ∠J = 50°.

15. A surveyor is on one side of a river. On the other side is a cliff of unknown height. To determine its height, the surveyor lays out a baseline AB of length 150m. From point A, she selects point C at the base of the cliff and measures $\angle CAB$ to be 51°. She selects point D on the top of the cliff directly above point C and measures an angle of elevation of 32°. She moves to point B and measures $\angle CBA$ as 62°. Find the height of the cliff, to the nearest tenth of a metre.



16. Sketch each angle in standard position and state the coordinates of P(x, y) on the unit circle, to three decimal places. Determine three other angles that are **<u>coterminal</u>** with it. Express at least one as a negative angle. Write the formula to determine any coterminal angle.
a) $\theta = 16°$ b) $\theta = -25°$ c) $\theta = 152°$ d) $\theta = 212°$ e) $\theta = 284°$

17. For each of the following angles: a) 150° b) 240° and c) 315°
i) Sketch each angle in standard positions.
ii) State the coordinates of point P(x, y) on the terminal arm of the unit circle using exact values.
iii) State the exact value of the trigonometric function for each of the 6 trig functions. Do not use a calculator. Use your special triangles.

18. Find the values to three decimal places of the 6 trig ratios for angle $\theta = 48°$.

19. Determine each value of θ for $0° \le \theta \le 360°$ to the nearest degree.
a) $\sin\theta = -0.227$ b) $\cos\theta = 0.567$ c) $\tan\theta = -1.543$
d) $\csc\theta = 4.039$ e) $\sec\theta = -1.745$ f) $\cot\theta = 0.975$

20. Prove the following trigonometric identities. State your reasoning where necessary (i.e. PTI, QTI, and RTI).
a) $\csc^2\theta - 1 = \csc^2\theta\cos^2\theta$
b) $\tan\theta + \cot\theta = \sec\theta\csc\theta$
c) $\dfrac{1}{1+\cos\theta} = \csc^2\theta - \dfrac{\cot\theta}{\sin\theta}$
d) $\dfrac{1+\csc\theta}{\cot\theta} - \sec\theta = \tan\theta$

21. Simplify the following rational expressions and state the restrictions on the variables.
a) $\dfrac{12ab-4b^2}{-6ab}$
b) $\dfrac{3m^2-m-10}{9m^2-25} \times \dfrac{4m^7+4m^6}{6m^6-12m^5}$
c) $\dfrac{2x^2-5x-3}{x-3} \div \dfrac{2x^2-11x-6}{x^2-3x}$
d) $\dfrac{5}{x-3} - \dfrac{3x}{x^2-4x+3}$

22. Solve the following quadratic equations using the most efficient method. Simplify radical answers. No decimals!
a) $y = 3x^2 - 15x + 18$
b) $y = x^2 + 2x - 7$
c) $y = 2x^2 - 11x - 21$
d) $y = 3x^2 - 4x - 5$

23. Determine the defining equation of the quadratic function that has zeros $1+2\sqrt{2}$ and $1-2\sqrt{2}$, and passes through the point (1, 8). **NOTE**: Express your final answer in standard form.

24. Determine the solution(s) to the following linear-quadratic systems.
a) $y = 2x - 5$ and $y = x^2 - 7x + 15$ b) $y = x + 6$ and $y = -2x^2 + 5x + 4$

25. What kind of line is the linear function in question #24b? Explain your answer.

26. Write the following powers as roots first. Then evaluate. Write answers as fractions in simplest form. No decimals!

a) 3^{-3} b) $27^{\frac{2}{3}}$ c) $\left(\dfrac{9}{4}\right)^{-\frac{1}{2}}$

27. Simplify the following powers to a single power (one base, one exponent).

a) $\left(a^{\frac{3}{4}}\right)\left(a^{\frac{1}{2}}\right)$ b) $\left(\dfrac{x^{\frac{1}{3}}}{x^{-2}}\right)^{\frac{1}{6}}$ c) $\dfrac{\left(n^{x+3y}\right)\left(n^{2x-4y}\right)}{\left(n^{3x-4y}\right)^{2}}$

28. The population of a colony of bacteria doubles every 10 hours. There are 200 bacteria at the beginning of the experiment.

a) Determine an equation to model the relationship where *A(t)* is the population of bacteria and *t* is the time from the beginning of the experiment, in hours.

b) What is the population 3 days after the start of the experiment? Round to the nearest whole number.

29. Complete the following table.

Sequence	Next 3 terms	Type of Sequence (A, G, N)	General Term (in simplified form)
11, 5, -1, -7, …			
$\frac{1}{3},\frac{2}{3},\frac{4}{3},\frac{8}{3},\ldots$			
$\frac{1}{2},\frac{3}{5},\frac{9}{8},\frac{27}{11},\ldots$			

30. State the first 4 terms for each of the following sequences.

a) $t_1 = 3, t_2 = 2, t_n = 5t_{n-1} - 4t_{n-2}$, $n > 2$ b) $f(n) = 2n^3 - n$

31. Draw Pascal's Triangle up to and including the 8th row.

32. Use Pascal's Triangle to expand the following binomial. Make sure your final answer is fully simplified.

a) $(a+b)^8$ b) $(m^4 + 2n)^6$

33. Calculate the following…

a) the amount of a $2500 investment after 7 years with an annual interest rate of 5.3%, compounded semi-annually.

b) the annual interest rate required for an investment of $4000 to double in 8 years if it is compounded monthly. Round to the nearest tenth of a percent.

c) the number of years required for a $2500 investment to grow to $4000 if interest is 7.2% p.a. compounded quarterly. Round answer to the nearest tenth of a year.

d) the amount you need to invest today in order to have $6000 in 5 years if interest is 7.8% p.a. compounded semi-monthly (twice a month).

34. Jack's parents invest $200 at the end of each month for the next 18 years for his education. The investment pays 5.1% p.a. compounded monthly.

a) How much will Jack have in his education fund at the end of the 18 years?

b) How much interest was earned on the investment?

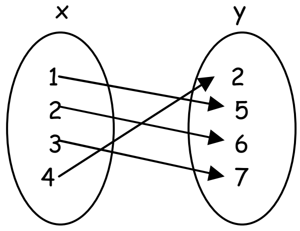
35. Determine the present value of a loan with payments of $500 every 6 months for 15 years if interest is 6.2% p.a. compounded semi-annually.

36. Sherri would like to have $12 000 after 5 and a half years. She will make a regular deposit every 3 months into an investment account that pays 5.2% p.a. compounded quarterly. Calculate the amount she will need to deposit every 3 months.

37. John has an investment of $45 000 that pays 6.3% p.a. compounded monthly. How much can he withdraw each month for the next 8 years?

MCR3U **EXAM REVIEW ANSWER KEY**

1) a) D: {2, 3, 4}
b) both f and h because there is only one y-value for each x-value
c) R: {2, 3, 4}
d) h(2) = 6
e) yes because each (x, y) pair in f is switched in g.
f) x = 3
g) $h^{-1}(2) = 4$
h)

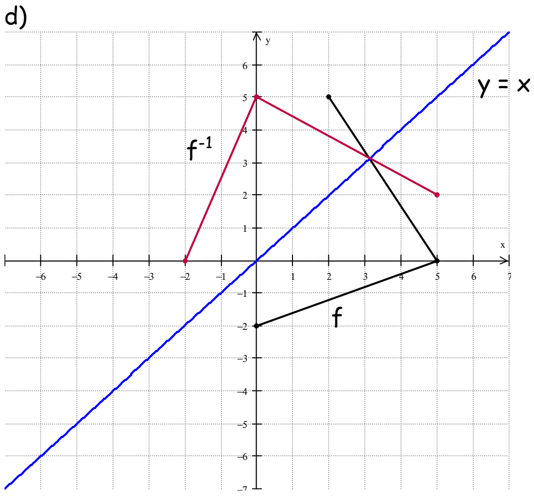


i) false

2) D: {x∈ R}, R: {y ≤ 6,y∈R} because it is a quadratic function where the parabola has a vertex at (1, 6) and is opening down , since a is negative.

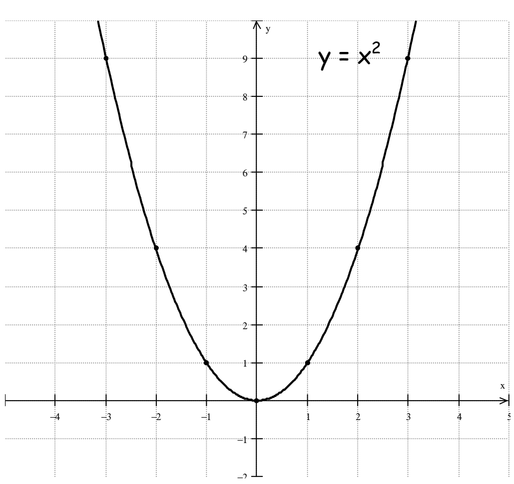
3) a) $f^{-1}(x) = \frac{x-9}{2}$, Original (f): D: {x∈ℜ}, R: {y∈ℜ} Inverse (f^{-1}): D: {x∈ℜ}, R: {y∈ℜ}
b) $f^{-1}(x) = (x + 5)^2 - 4$, Original (f): D: {x ≥ -4, x∈ℜ}, R: {y ≥ -5, y∈ℜ},
Inverse (f^{-1}) D: {x ≥ -5, x∈ℜ}, R: {y ≥ -4, y∈ℜ}

4) a) Not a function because it does not pass the Vertical Line Test
b) f(0) = -2
c) x = 2
c) D:{x∈ℜ, 0 ≤ x ≤ 5}, R:{y∈ℜ, -2 ≤ y ≤ 5}
d)

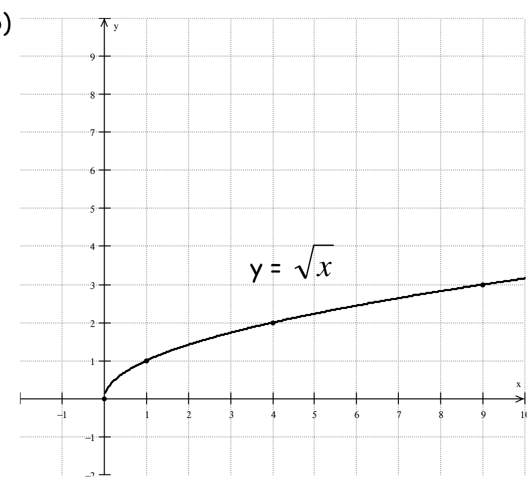


5) a) g(-3) = 12 b) $g(x - 3) = x^2 - 6x + 12$ c) f(7) = 13

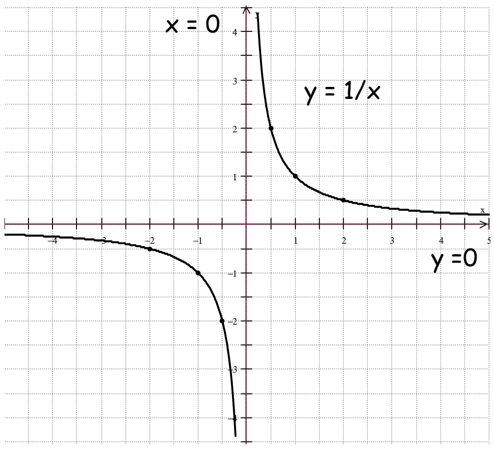
6) a)



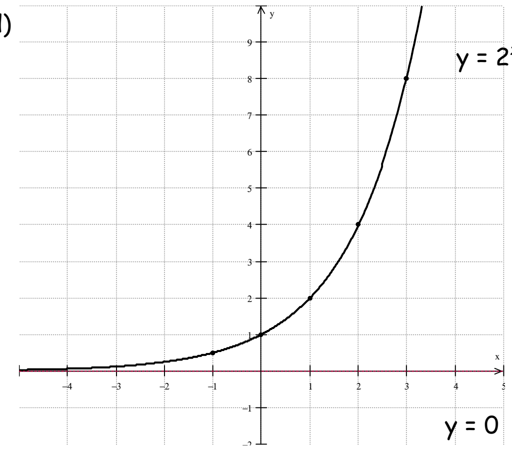
b)



c)



d)

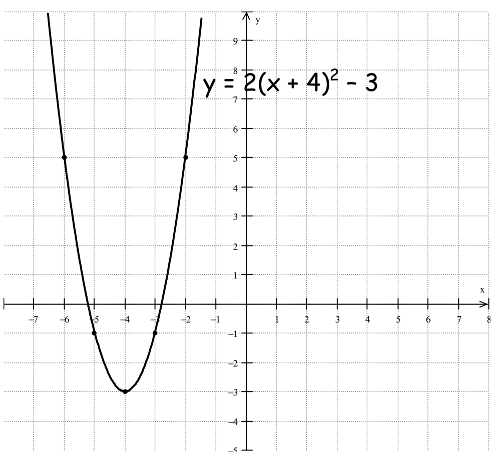


7) a) vertical stretch by a factor of 3, reflection in the y-axis, or horizontal reflection, horizontal compression by a factor of $\frac{1}{2}$, and translation of 4 units right and 11 units up.

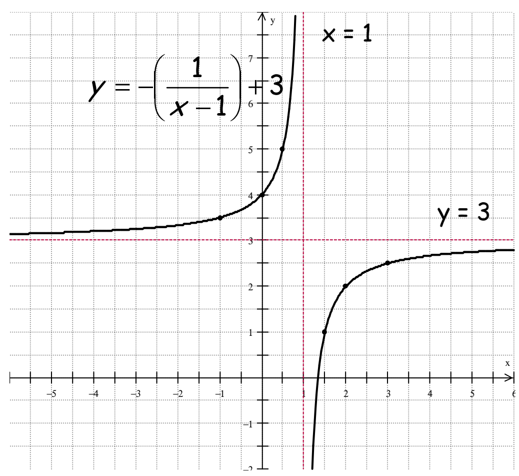
b) reflection in x-axis, or vertical reflection, vertical compression by a factor of 1/5, horizontal stretch by a factor of 2, and translation of 2 units left and 9 units down.

8) a) x = 0 and y = 0 b) x = -5 and y = -6 c) x = 8 and y = 7 d) y = 0 e) y = -9 f) y = 12

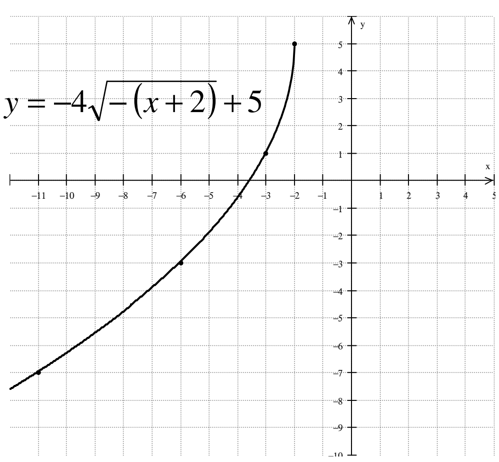
9) a)



b)



c)



10) y = 3sin[-(x – 2)] + 4

11)

Function	Amp	Period	Max y	Min y	Phase shift	Vertical Shift	Domain	Range
$y = \sin\theta$	1	360°	1	-1	0°	0	{θ∈ℜ}	{y∈ℜ, -1 ≤ y ≤ 1}
$y = 3\sin\left[\frac{1}{2}(\theta - 60°)\right] - 2$	3	720°	1	-5	60°	-2	{θ∈ℜ}	{y∈ℜ, -5 ≤ y ≤ 1}
$y = -1.5\sin[3(\theta + 240°)] + 5.5$	1.5	120°	7	4	-240°	5.5	{θ∈ℜ}	{y∈ℜ, 4 ≤ y ≤ 7}
$y = 5\cos[4(\theta - 22°)] + 8$	5	90°	13	3	22°	8	{θ∈ℜ}	{y∈ℜ, 3 ≤ y ≤ 13}
$y = -2\sin[5(\theta + 40°)] + 15$	2	72°	17	13	-40°	15	{θ∈ℜ}	{y∈ℜ, 13 ≤ y ≤ 17}
$y = -8\cos\left[\frac{1}{5}\theta\right] - 11$	8	1800°	-3	-19	0°	-11	{θ∈ℜ}	{y∈ℜ, -19 ≤ y ≤ -3}

12) a) Maximum: 28m and minimum: 9m b) amp: 9.5, period: 13.6, v. displacement: 18.5

c) For a cosine curve, the first max represents one possible phase shift, so d = 6,

$\therefore$ a possible equation is $y = 9.5\cos\left[\dfrac{360°}{13.5}(x-6)\right] + 18.5$

13) a) Max: 28m, min: 4m, $\therefore$ a = 12 and c = 16, p = 110 – 35 (distance b/w 2 maximums), $\therefore$ k = 360°/75
= 75 = 4.8

and since this is a cosine function and the first max occurs at 35h, then d = 35

$\therefore$ a possible equation for this function is: $h = 12\cos[4.8(t-35)] + 16$

b) $\doteq$ 11.8m

14) a) b $\doteq$ 11.5cm, $\angle A \doteq 41°$, $\angle C \doteq 30°$

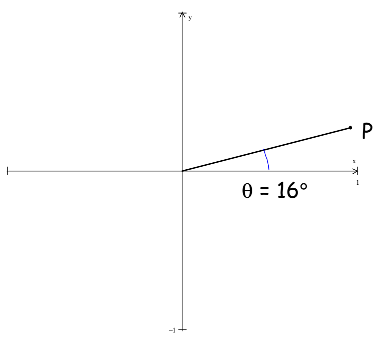
b) Δ#1: $\angle F_1 \doteq 41°$, $\angle E_1 \doteq 117°$, $e_1 \doteq 7.6$cm, and
Δ#2: $\angle F_2 \doteq 139°$, $\angle E_2 \doteq 19°$, $e_2 \doteq 2.8$cm

c) $\angle I = 93°$, g $\doteq$ 2.0cm, h $\doteq$ 4.2cm

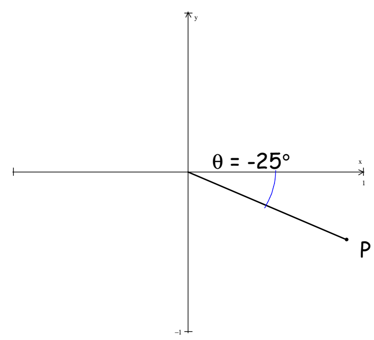
d) Δ#1: $\angle L_1 \doteq 73°$, $\angle K_1 \doteq 57°$, $k_1 \doteq 4.4$cm, and
Δ#2: $\angle L_2 \doteq 107°$, $\angle K_2 \doteq 23°$, $k_2 \doteq 2.0$cm

15) h $\doteq$ 89.9m

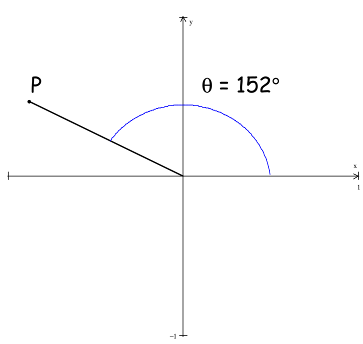
16) a) P(0.961, 0.276),
Coterminal angles: θ = 376°, 736°, 1096°, ...
or -344°, -704°, -1064°, ...
Any coterminal angle: θ = 16°+ 360°n



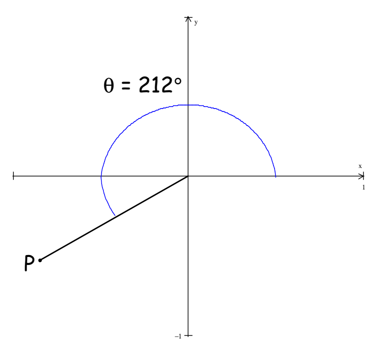
b) P(0.906, -0.423),
Coterminal angles: θ = 335°, 695°, 1055°, ...
or -385°, -745°, -1105°, ...
Any coterminal angle: θ = -25°+ 360°n



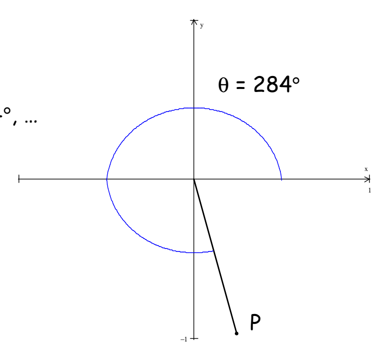
c) P(-0.883, 0.469),
Coterminal angles: θ = 512°, 872°, 1232°, ...
or -208°, -568°, -928°, ...
Any coterminal angle: θ = 152°+ 360°n



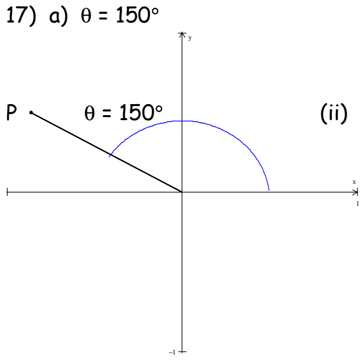
d) P(-0.848, -0.530),
Coterminal angles: θ = 572°, 932°, 1292°, ...
or -148°, -508°, -868°, ...
Any coterminal angle: θ = 212°+ 360°n



e) P(0.242, -0.970)
Coterminal angles: θ = 644°, 1004°, 1364°, ...
or -76°, -436°, -796°, ...
Any coterminal angle: θ = 284°+ 360°n



17) a) θ = 150°

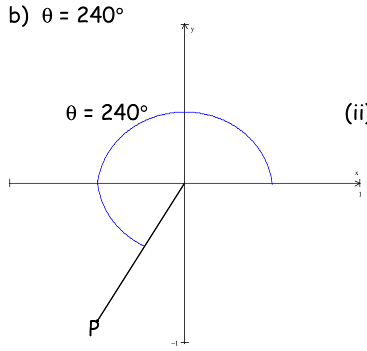


(ii) $P\left(-\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$

(iii) sin 150° = $\dfrac{1}{2}$, cos 150° = $-\dfrac{\sqrt{3}}{2}$, tan 150° = $-\dfrac{1}{\sqrt{3}}$

csc150° = 2, sec150° = $-\dfrac{2}{\sqrt{3}}$, cot150° = $-\sqrt{3}$

b) θ = 240°

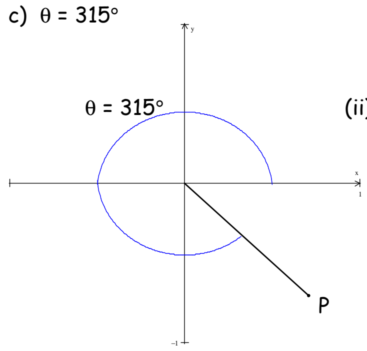


(ii) $P\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$

(iii) sin240° = $-\frac{\sqrt{3}}{2}$, cos240° = $-\frac{1}{2}$, tan240° = $\sqrt{3}$

csc240° = $-\frac{2}{\sqrt{3}}$, sec240° = -2 , cot240° = $\frac{1}{\sqrt{3}}$

c) θ = 315°



(ii) $P\left(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$

(iii) sin315° = $-\frac{1}{\sqrt{2}}$, cos315° = $\frac{1}{\sqrt{2}}$, tan315° = -1

csc315° = $-\sqrt{2}$, sec315° = $\sqrt{2}$, cot315° = -1

18) sin 48° ≐ 0.743, cos 48° ≐ 0.669, tan 48° ≐ 1.111, csc 48° ≐ 1.346, sec 48° ≐ 1.494, cot 48° ≐ 0.900

19) a) θ ≐ 193°, 347° b) θ ≐ 55°, 305° c) θ ≐ 123°, 303° d) θ ≐ 14°, 166° e) θ ≐ 125°, 235° f) θ ≐ 46°, 226°

20) Answers may vary. See teacher to check your proofs.

21) a) $-\frac{2(3a-b)}{3a}, a \neq 0, b \neq 0$ b) $\frac{2m(m+1)}{3(3m-5)}, m \neq \pm\frac{5}{3}, 0, 2$

c) $\frac{x(x-3)}{x-6}, x \neq -\frac{1}{2}, 0, 3, 6$ d) $\frac{2x-5}{(x-3)(x-1)}, x \neq 1, 3$

22) a) x = 2, 3 b) x = $-1 \pm 2\sqrt{2}$ c) x = 7, $-\frac{3}{2}$ d) x = $\frac{2 \pm \sqrt{19}}{3}$

23) y = $-x^2 + 2x + 7$

24) a) (4, 3) and (5, 5) b) (1, 7)

25) It is a **tangent** line because there is only one point of intersection between a tangent line and a quadratic. (i.e. There is one solution to the linear-quadratic system.)

26) a) $\left(\frac{1}{3}\right)^3 = \frac{1}{27}$ b) $\left(\sqrt[3]{27}\right)^2 = 9$ c) $\sqrt{\left(\frac{4}{9}\right)} = \frac{2}{3}$

27) a) $a^{\frac{5}{4}}$ b) $x^{\frac{7}{18}}$ c) n^{-3x+7y}

28) a) $A(t) = 200(2)^{\frac{t}{10}}$ b) A(72) ≐ 29407

29)

Sequence	Next 3 terms	Type of Sequence (A, G, N)	General Term (in simplified form)
11, 5, -1, -7, …	-11, -15, -19, -23, …	Arithmetic	$t_n = 11 + (n-1)(-4)$ $t_n = -4n + 15$
$\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, \frac{8}{3}, \ldots$	$\frac{16}{3}, \frac{32}{3}, \frac{64}{3}, \frac{128}{3}, \ldots$	Geometric	$t_n = \frac{1}{3}(2)^{n-1}$
$\frac{1}{2}, \frac{3}{5}, \frac{9}{8}, \frac{27}{11}, \ldots$	$\frac{81}{14}, \frac{243}{17}, \frac{729}{20}, \frac{2187}{23}, \ldots$	Neither Numerator: Geometric and Denominator: Arithmetic	$t_n = \frac{1(3)^{n-1}}{2 + (n-1)(3)}$ $t_n = \frac{3^{n-1}}{3n-1}$

30) a) 3, 2, -2, -18, … b) 1, 14, 51, 124, …

31)

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

32) a) $a^8 + 8a^7b + 28a^6b^2 + 56a^5b^3 + 70a^4b^4 + 56a^3b^5 + 28a^2b^6 + 8ab^7 + b^8$

b) $m^{24} + 12m^{20}n + 60m^{16}n^2 + 160m^{12}n^2 + 240m^8n^4 + 192m^4n^5 + 64n^6$

33) a) $3605.50 b) 8.7% c) 6.6 years d) $4064.91

34) a) $70 560.48 b) $27 360.48

35) $9674.74

36) $474.68

37) $597.96